AP Calculus

Brief Description of the Course:

Our study of calculus is divided into two major topics: differential and integral calculus.

Differential calculus enables us to calculate rates of change, determine optimization of real-life problems, to find the slope of a curve, and to calculate velocities and accelerations of moving bodies. Integral calculus is used to find the area of an irregular region in a plane, to measure lengths of curves, determine the volume of a rotational solid, and to calculate centers of mass of arbitrary solids.

Most AP Calculus students enter this course with the knowledge of functional relationships. The task is to perfect each student's understanding of functional applications while developing the theories of derivatives and integrals. Through additional instruction, students will develop an understanding of the application of derivatives and integrals in solving problems. Technology will be used as a means of developing a multi-representational approach to the study of functions.

Throughout the course, students will discuss the Calculus AB exam. As part of the curriculum, appropriate multiple choice and free response questions will be practiced weekly. Effective study techniques will be reviewed as part of each unit assessment. Students will create a comprehensive study guide to aid in preparation for the exam in May. All exams will be modeled after the advanced placement exam.

Class Profile:

We provide 2 sections of Calculus AB each year in addition to at least one section at the Honors level. Most of the students participating in Calculus began their study of Algebra in the 8th grade. However there is a small sampling of students who began studying Algebra in grade 9 and accelerated their mathematics curriculum by studying Geometry and Algebra 2 in the sophomore years. All students participating in AP Calculus must meet the required prerequisite of a 90-100% average in PreCalculus Honors.

Teaching Strategy:

Although in AP Calculus I find myself working with some of the best math students my

school has, I find that they lack a clear understanding of functions (including trig, logarithmic, and exponential) at the level necessary to participate in an AP Calculus curriculum. For this reason, we will spend two weeks applying the Rule of Four (understanding functions "graphically, verbally, numerically, and algebraically") while developing a deeper understanding of the use of technology prior to beginning our Calculus curriculum. This is a very trying time for many students as they are forced to examine functions at levels they may not feel comfortable at and it is then that we are able to address the commitment necessary in studying AP Calculus.

Throughout our curriculum we continue to present each topic numerically, verbally, graphically, and algebraically. New topics are presented in an interactive lecture style. I, as the instructor, present a new concept or topic and the students openly discuss the idea, asking questions, working on problem sets, and using each other to confirm their work. Students make extensive use of the TI-83 calculator (a class set is maintained, however each student is strongly urged to provide their own for home study), a TI Viewscreen and SmartBoard technology.

Beginning with the study of derivatives, students are assigned multiple choice questions from previous editions of the placement exam on their tests and quizzes. Students are allowed to use a calculator on any question in which it was allowed on the original exam. All tests are cumulative and students are responsible for any material covered from Algebra 1 through the day of the test. Tests are 2 periods in length, one with calculator and one without. A free response question from a previously released test is included on the appropriate day. In addition to the test free response question, students also receive 3 practice free response questions each Monday as a written journal assignment (due Thursday). On Friday, students work in small groups evaluating their classmates' anonymous responses using the published grading standard. Each group reports out to the class, discussing the question and the standard by which it is graded, common errors, best practices, and grading requirements commonly implemented at the AP reading. I grade the written journal questions as an additional quiz grade.

In addition to written journal assignments comprised of AP free response questions, students are assigned various activities modeling the use of mathematics in real life. The activities have been chosen for their ability to reinforce the Rule of Four. Students complete most of the activities outside of class however some are done cooperatively during class time.

I teach my course "to the test" and in doing so feel that students are being prepared for the exam throughout all their class-related activities. Students are provided 3 full length

practice exams at each of their school vacations. In addition, I offer an extra-help/study session one night each week for 3 hours. Calculus students are encouraged to attend this session following the winter recess in February to begin preparation for the exam.

Evaluation:

Quarter grades are computed considering homework, tests, and quizzes and weighted as required by the mathematics department chair. Homework contributes 20% of the quarter grade. One assignment is checked each week (unannounced) and the grade assigned to each assignment for the week. Tests and unit assessments constitute 45% of the quarter grade. Quizzes are weighted 35% of the quarter average. Projects, journals, portfolios and notebook checks will be addressed in Classroom Management Plans and assigned as Test or Quiz grades.

Personal Statement:

In accepting the position of AP Calculus teacher in my school district I chose to attend the AP Institute sponsored by Collegeboard. The experience was enlightening, informative, yet overwhelming. After teaching my first year, I chose to attend the same institute again and found that I was able to absorb everything I missed my first time attending the institute, further refining my curriculum.

I. Review of Functions (approx. 2 weeks):

Students will understand that functions are tools for describing the real world in mathematical terms. They will further understand the particular importance of functions through the behaviors they describe. Students will become familiar with technology and its use in interpreting results

Unit 1: Review of Functions:

A. Graphing
    1. Intercepts, Symmetry, Intersection points
    2. Using graphing calculator to consider multiple perspectives

B. Lines and Linear Models
    1. Slope as a rate of change
    2. Graphing Linear Models
    3. Equations of Lines

C. Functions
    1. Functions and Functional Notation
    2. Domain and Range of a Function
    3. Eight Basic Functions and their Transformations
    4. Piecewise Functions
    5. Composition of a Function

D. Exponential and Logarithmic Functions
    1. Exponential Growth and Decay
    2. Inverse Functions
    3. ACTIVITY: Testing for Inverses Graphically
    4. Logarithmic Functions
    5. Properties of Logarithms

E. Trigonometric Functions
    1. Graphs of Basic Trig Functions
        i. Domain and Range
        ii. Transformations
        iii. Inverse Trig Functions
    2. Applications: The Ferris Wheel Project

F. Fitting Models to Data
    1. Least Squares Regression Linear
    2. Least Squares Regression Quadratic
    3. Trig Models

4. ACTIVITY: Height vs Hand Size
5. ACTIVITY: Exponential Decay of M & M's
6. ACTIVITY: The Tides of Cape Cod

II. Limits (approx. 2 weeks)

Students will identify the limiting values of functions by using straightforward calculations, graphical investigation, as well as analysis of data. Students will work toward using limits to investigate the continuity of a function and apply their analysis to real life situations.

Unit 2: Limits and Their Properties:

A. Limits Graphically and Numerically
    1. Evaluating Limits
    2. Definition of a Limit
    3. Limits that Fail to Exist
B. Evaluating Limits Analytically
    1. Properties of Limits
    2. Strategies for Finding Limits
    3. Cancellation and rationalization Techniques
    4. Squeeze Theorem
    5. Special Trig Limits
    6. ACTIVITY: Graphs and Limits of Trig Functions
C. Continuity and One-Sided Limits
    1. Continuity at a Point and Open Interval
    2. Continuity on the Closed Interval
        a. One sided vs 2 sided limits
    3. Discontinuous Functions
        a. Removable vs Non Removable
    4. Properties of Continuity
    5. Intermediate Value Theorem
D. Infinite Limits
    1. Asymptotic Behavior

2. End Behavior

E. Rates of Change

III. Differentiation: (approx. 3 weeks)

Students will use limits to find the slopes of tangent lines to a graph at a point and develop the process of differentiation. The student will expand the derivative to involve the rate of change of one variable with respect to another and apply this to production or growth rates and changes in motion, specifically velocity and acceleration. An in-depth analysis of the position function will be explored. Students will finish the unit with the study of change with respect to time or related rates.

A. The Derivative and the Tangent Line Problem
    1. Limits of Secant Lines to a Curve
    2. Derivatives that fail to exist
    3. Differentiability implies Local Linearity and Continuity
    4. ACTIVITY: Seeing Differentiability (Testing Local Linearity)

B. Basic Differentiation and Rates of Change
    1. Average vs Instantaneous rates of Change
    2. Particle Motion
    3. ACTIVITY: Hypertext Investigation into Speed and Velocity

C. Product and Quotient Rules and Higher Order Derivatives
    1. Application of the Product and Quotient rules

2. Simple Trig Functions
3. Higher order derivatives

D. The Chain Rule
   1. Composite functions
   2. Trig functions requiring the Chain Rule

E. Implicit Differentiation
   1. Distinquish between Implicitly expressed and Explicitly expressed functions
   2. Use Implicit techniques to find derivatives
   3. ACTIVITY: Optical Illusion

F. Related Rates
   1. ACTIVITY: Applications package

IV. Applications of the Derivative: (approx. 4 weeks)

Students will use the derivative to reach various conclusions about the maximums and minimums of functions, concavity of curves, as well as points of inflection. Students will also use the graph of a function's derivative to determine the shape of the original function and, with one point on f(x), generate the actual curve. Students will model differential calculus in optimization examples from business, industry, engineering, and economics and use their knowledge of the derivative to solve associated problems.

A. Extrema on an Interval
   1. Critical numbers
   2. Definition of Extrema on an Interval
   3. Understanding of Relative vs Absolute Extrema

4. Finding extrema on a closed interval

B. Rolle's Theorem and the Mean Value Theorem
    1. Understanding and applying Rolle's Theorem
    2. Understanding and applying the Mean Value Theorem

C. Increasing/Decreasing Function and the First Derivative Test
    1. Determining the intervals over which a function is Increasing or Decreasing
    2. Application of the First Derivative Test

D. Concavity and the Second Derivative Test
    1. Determining the intervals over which a function is Convcave up or down
    2. Determining points of inflection of a function
    3. Finding extrema through the application of the Second Derivative Test
    4. ACTIVITY: Exploring changes in the 2^{nd} derivative

E. Limits at Infinity
    1. Determining limits at infinity
    2. Using limits at infinity to find horizontal asymptotes of a function (as applicable)

F. Newtons Method
    1. Approximation of zeros of irregular functions

G. Curve Sketching
    1. Applying conclusions of the first and second derivative in analyzing the graph of a function
    2. Incorporation of other graphing characteristics in sketching curves without the necessity of technology
    3. ACTIVITY: MathGraphs 65G; Analyzing graphs of the 1^{st} and 2^{nd} derivative

H. Applications
    1. Solving maximum/minimum problems through the use of optimization techniques.
    2. The use of differentials in making tangent line approximations and estimations of the values of functions
    3. Error Propagation and the estimation of percent as well as relative error in real life applications
    4. ACTIVITY: Applications Package

V. AntiDerivatives and Integration: (approx. 4 weeks)

Students will apply the theory of finite sums in the accumulation of rates of change to realize the area under a curve and the process of anti-differentiation or integration. Students will investigate the connections between differentiation and integration in analyzing the Fundamental Theorem of Calculus. Students will explore various methods of evaluating integrals of functions, including graphical analysis, various approximation and summation techniques, as well as through algorithms and the use of technology.

A. Anti-Derivative and Indefinite Integrals
    1. The general solution of a differential equation
    2. Indefinite notation of integrals
    3. Finding antiderivatives using basic integration rules
    4. Particular solutions of a differential equation
B. Area
    1. Sigma Notation, summation formulas, and the evaluation of sum
    2. Area under a curve, its approximation and meaning
    3. Using limits to determine the area under a curve
C. Reimann Sums and the Definite Integrals
    1. Definition of the Reimann Sum
    2. Evaluate a definite integral using the limit technique
    3. Evaluate definite integrals using the properties of definite integrals
D. First Fundamental Theorem of Calculus
    1. Evaluating definite integrals using the Fundamental Theorem of Calculus
    2. Analysis and use of the Mean Value Theorem of Integrals
    3. Average Value of a Function over an interval and its connection to the Mean Value Theorem
    4. Analysis and use of the Second Fundamental theorem of Calculus
    5. ACTIVITY: Demonstrating the Fundamental Theorem of Calculus
E. Integration Techniques
    1. Pattern Recognition

2. Change of Variables
3. Power Rule
4. Trapezoidal and Simpson's Rule and the approximation of their errors
5. ACTIVITY: Calculating Real Life Area using the Trap Rule

VI. Differentiation and Integration of Logarithmic, Exponential, and Transcendental Functions: (approx. 4 weeks)

Students will use their knowledge of the definition of the inverse as the reflection of a function about the line y = x to make assumptions about the differentiability of various functions and their inverses. The investigation into inverse relationships will be expanded to cover functions that have bases other than the natural number 'e' or the natural logs of base 10. Students will examine the Law of Exponential Change, Newton's Law of Cooling, and other exponential and logistic growth/decay models such as compounded interest, radioactivity, and population examples.

A. Differentiation of the Natural Log Function
    1. Properties of Natural Logarithmic Functions
    2. Definition of the Number "e"
    3. Derivative techniques for the Natural Log Function
B. Integration of the Natural Log Function
    1. The Log Rule of Integration for integration of a Rational Function
    2. Integration of Trigonometric Functions
C. Inverse Functions
    1. Investigating Inverse Functions
    2. Derivatives of Inverse Functions and their relationships
D. Differentiation and Integration of Exponential Functions
    1. Developing properties of natural exponential functions
    2. Differentiation and integration of natural logarithmic functions
    3. ACTIVITY:
E. Using Functions with Bases of Other than e.
    1. Definition of functions with bases other than "e"

2. Differentiation and integration of exponential functions with bases other than “e”
3. Models utilizing bases other than “e”
4. ACTIVITY: Developing Formula for Differentiatiion

F. Differentiation and Integration of the Inverse Trig Functions
1. Properties of six trigonometric functions
2. Differentiation and integration of inverse trig functions
3. ACTIVITY: Using Graphing Utilities to Estimate Slope

G. Introduction to Hyperbolic Functions
1. Properties and definitions of hyperbolic functions
2. Differentiation and integration techniques for hyperbolic functions
3. Properties of inverse Hyperbolic functions
4. Differentiation and integration of inverse hyperbolic functions
5. ACTIVITY: The Tractix Problem
6. ACTIVITY: The St Louis Arch

VII. Differential Equations: (Approx. 3 weeks)

Students will analyze slope fields and determine its relevance to families of graphs and general solutions to differential equations. Students will develop and use Euler’s Method as a numerical approach to approximating the particular solution of a differential equation. Students will employ the method of separation of variables and develop strategies for the use of the separation technique in problem-solving. In furthering their understanding of exponential growth and decay models, students will solve first order differential equations and develop the use of the integrating factor in the separation of variables technique and its application in Bernoulli’s Theorem and Newton’s Law of Cooling.

A. Slope Field’s and Euler’s Method
1. Using initial conditions to find particular solutions
2. Using Slope Fields to approximate solutions
3. Using Euler’s Method to approximate solutions

B. Differential Equations: Growth and Decay
1. Separation of Variables to solve simple differential equations
2. Modeling growth and decay with exponential functions
3. ACTIVITY: Applications Package Modeling Growth and Decay

C. Separation of Variables
    1. Recognizing differential equations that use separation of variables technique in solving
    2. Solving homogeneous differential equations
    3. Model and solve problems using differential equations
    4. ACTIVITY: Project Weight Loss

D. First Order Linear Differential Equations
    1. Solving first order differential equations
    2. Introduce Bernoulli differential equation
    3. Use differential equations in problem-solving

VIII. Applications of Integration: (approx. 4 weeks)

Students will use the technique of Reimann Sums as a model to write a definite integral which can be used to study the net change of an irregular region. In developing the models necessary for integration, students will analyze methods of using integration in determining rotational volume, area of the regions that exist between curves, as well as other physical properties that are affected by displacement such as work, centers of gravity, and fluid forces and pressures.

A. Area of Region between Two Curves
    1. Integration as a process of accumulation

2. Using integration to find the area between 2 curves
3. Using integration to find the area between intersecting curves
4. ACTIVITY: Modeling Data-Social Security Trust Fund

B. Volume of Rotational Solids
1. Using the Disk and Washer Methods
2. Using the Shell Method
3. Comparison of the Disk and the Shell Methods

C. Arc Length and Surface Areas created by Revolution
1. Find the arc length of a smooth curve
2. Find the area of a surface of revolution
3. ACTIVITY: Modeling Data-Property Area

D. Work
1. Work by a constant force
2. Application of the integral to find work done by a variable force
3. ACTIVITY: Tidal Energy

E. Moments and Centers of Mass
1. Definition of Mass
2. Centers of Mass in one dimensional, two dimensional, and planar regions

F. Fluid Pressure and Fluid Force
1. Definition of fluid pressure
2. Definition of fluid force.
3. Application of the integral to fluid pressure and force
4. ACTIVITY: The Great Molasses Disaster of 1919

IX.Improper Integrals and Other Integration Techniques (3 weeks)

Having used integration by substitution previously as a method of finding the ant-

derivative of a chain rule, students will develop integration by parts to account for the product rule of differentiation, trigonometric substitutions to integrate the square roots of polynomial functions, and partial fraction integration to allow for the integration of rational functions. Students will develop recognition of classes of integration techniques and employ tables of integration as supplied in their text. Finally, students will return to the study of infinite integrals through L'Hospital's Rule and further investigate integration where one limit of integration is infinite or one integrand is unbounded.

A.Basic Integration Rules

1. Review Basic Integration procedures

E. Integration by Parts

1. Using integration by parts to find an integral

F. Trigonometric Integrals

1. Using Trigonometric substitution to solve integrals
2. Use Trig integrals to model real-life problems
3. Use Trig substitution to solve integrals
4. Using Trig substitution to solve real-life problems

G. Partial Fractions

1. Understanding Partial Fraction Decomposition
2. Using Partial Fraction Decomposition with linear factors for solving integrals
3. Strategies for using quadratic factors in Partial fraction decomposition

H. Miscellaneous Integrals

1. Integration by Tables
2. L'Hospital's Rule and Indeterminate Forms
3. Improper Integrals
4. ACTIVITY: Exploring L'Hospital's Rule Graphically

PRINCIPLE TEXT:

Larson, Roland E., Hostetler, Robert P., and Edwards, Bruce H.
Calculus of a Single Variable. 6th Edition, Houghton-Mifflin Company, 1998

Supplementary Material:

Larson, Roland E., Hostetler, Robert P, and Edwards, Bruce H.
Calculus of a Single Variable. 8th Edition. Houghton-Mifflin Company. 2006.

Finney, Ross L., Demana, Franklin D., Waits, Bert K., Kennedy, Daniel.
Calculus: Graphical, Numerical, Algebraic. 3rd edition. Pearson Education, Inc. as Pearson Prentis-Hall Publishing. 2007.

Hughes-Hallett, Deborah, Gleason, Andrew, McCallum, William, et al.
Calculus Single and MultiVariable. John Wiley and Sons, Inc. 2005.

AP Calculus Multiple-Choice Question Collection 1969-1998. Collegeboard Products. 23 August 2006.
http://apcentral.collegeboard.com/apc/public/courses/teachers_corner/2178.html

AP Calculus Free-Response Question Collections. Collegeboard Products. 23 August 2006.
<http://apcentral.collegeboard.com/apc/public/courses/teachers_corner/2178.html>

Technology:

Texas Instruments Graphing Calculator. Model TI-84 Plus.